1. The binomial expansion of $\left(2x + \frac{5}{x}\right)^6$ has a term which is a constant. Find this term. [4]

2. Find the coefficient of x^3 in the binomial expansion of $(2 - 4x)^5$. [4]

3. Find the coefficient of x^4 in the binomial expansion of $(5 + 2x)^7$. [4]

4. Find the binomial expansion of $(1 - 5x)^4$, expressing the terms as simply as possible. [4]

5. Find the coefficient of x^4 in the binomial expansion of $(x - 3)^5$. [3]

6. Expand $(2x - 3)^5$, writing each term in its simplest form. [4]

7. You are given that, in the expansion of $(a + bx)^5$, the constant term is 32 and the coefficient of x^3 is -1080.
Find the values of a and b. [5]

8. Find the constant term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{15}$. [2]

9. Find the binomial expansion of $(3 - 2x)^3$. [4]

10. Find the term in x^3 in the binomial expansion of $(2 + x)^5$. [2]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			identifying term as $20(2x)^3\left(\frac{5}{x}\right)^3$ oe	M3	condone lack of brackets; M1 for $[k](2x)^3\left(\frac{5}{x}\right)^3$ soi (eg in list or table), condoning lack of brackets and M1 for $k = 20$ or eg $\frac{6\times5\times4}{3\times2\times1}$ or for 1 6 15 20 15 6 1 seen (eg Pascal's triangle seen, even if no attempt at expansion) and M1 for selecting the appropriate term (eg may be implied by use of only $k = 20$, but this M1 is not dependent on the correct k used) or B4 for 20 000 obtained from multiplying out $\left(2x+\frac{5}{x}\right)^6$ allow SC3 for 20000 as part of an expansion	xs may be omitted; eg M3 for 20 × 8 × 125 first M1 not earned if elements added not multiplied; otherwise, if in list or table bod intent to multiply M0 for binomial coefficient if it still has factorial notation may be gained even if elements added
			20000	A1	**Examiner's Comments** A large proportion of candidates did not understand what was meant by 'a term which is constant'. A good number still found the term $20(2x)^3\left(\frac{5}{x}\right)^3$ but did not recognise it as the term needed to find the constant. Even those who did know what was meant by a constant term usually wrote out the whole expansion rather than identifying which was the relevant term from the start. Brackets were often missing, leading to incorrect evaluations.	
			Total	4		
2			−2560 www	4	B3 for 2560 from correct term (NB coefficient of x^4 is 2560) or B3 for neg answer following 10 × 4 × −64 and then an error in multiplication	ignore terms for other powers; condone x^3 included; but eg 10 × 4 × −64 = 40 − 64 = −24 gets M2 only

					or M2 for $10 \times 2^2 \times (-4)^3$ oe; must have multn signs or be followed by a clear attempt at multn; or M1 for $2^2 \times (-4)^3$ oe (condone missing brackets) or for 10 used or for 1 5 10 10 5 1 seen for those who find the coefft of x^2 instead: allow M1 for 10 used or for 1 5 10 10 5 1 seen; and a further SC1 if they get 1280, similarly for finding coefficient of x^4 as 2560 **Examiner's Comments** Finding the binomial coefficient was done successfully by many candidates, but a surprising number omitted the negative sign in their answer. Virtually all the candidates managed to pick up at least one mark, usually for writing down the binomial coefficient either in Pascal's triangle or as part of an expression. Many candidates wrote down an expression involving the key elements 10, 2^2 and $(-4)^3$, though the brackets were often omitted. It was at this point that some arithmetical errors crept in, in the attempts to calculate $10 \times 4 \times -64$.	condone missing brackets eg allow M2 for $10 \times 2^2 \times -4x^3$ ${}^5C_3$ or factorial notation is not sufficient but accept $\frac{5\times4\times3\times2\times1}{2\times1\times3\times2\times1}$ oe 10 may be unsimplified, as above M1 only for eg 10, 2^2 and $-4x^3$ seen in table with no multn signs or evidence of attempt at multn [lack of neg sign in the x^2 or x^4 terms means that these are easier and so not eligible for just a 1 mark MR penalty]
			Total	4		
3			70 000 www	4	throughout, condone xs included eg $(2x)^4$ M3 for $35 \times 5^3 \times 2^4$ oe or M2 for two of these elements multiplied or M1 for 35 oe or for 1 7 21 35 35 21 7 1 row of Pascal's triangle seen	annotate this question if partially correct allow 4 for $70\,000x^4$ www; may also include other terms in expansion. Allow marks even if wrong term selected; mark the coefficient of x^4 may be unsimplified, but do not allow 35 in factorial form unless evaluated later or for all three elements seen together (eg in table) but not multiplied

					Examiner's Comments Many candidates were able to establish the desired product of $35 \times 5^3 \times 2^2$ in finding the binomial coefficient. There were fewer failing to cope correctly with $(2x)^4$ than in similar past questions on this topic. However, few candidates were confident enough with their number bonds, or quick mental methods such as repeated doubling, to realise that $5^3 \times 2^4$ or 125×16 could be easily evaluated as 2000. So they often attempted 35×125 etc with a distinct lack of success.	
			Total	4		
4			$1 - 20x + 150x^2 - 500x^3 + 625x^4$ as final answer	4	part marks can be awarded for earlier stages if final answer incorrect or not fully simplified: **M3** for 4 terms correct or for all coefficients correct except for sign errors or for correct answer seen then further ‘simplified’ or for all terms correct eg seen in table but not combined (condone eg $+(-20x)$ or $+(-20x$ instead of $-20x$) **M2** for 3 terms correct or for correct expansion seen without correct evaluation of coefficients [if brackets missing in elements such as $(-5x)^2$ there must be evidence from calculation that $25x^2$ has been used] binomial coefficients such as ${}^4C_2$ are not sufficient – must show understanding of these symbols by at least partial evaluation; or **M1** for 1 4 6 4 1 soi, eg in Pascal's triangle or in expansion where powers of 5 have been ignored	for binomial coefficients, ${}^4C_2$ or factorial notation is not sufficient but accept $\frac{4\times3\times2\times1}{2\times1\times2\times1}$ oe etc any who multiply out instead of using binomial coeffts: look at their final answer and mark as per main scheme if 3 or more terms are correct, otherwise M0 **Examiner's Comments** Binomial expansion was done well in comparison with previous years. Most candidates remembered to use the correct coefficients and were comfortable multiplying them with powers of 5. There were not too many arithmetic errors.
			Total	4		
5			–15	B3(AO1.1) (AO1.1)	**B2** for 15 or $5\times(-3)^1$ or better | Do not accept ${}_5C_4$ as a correct	

				(AO1.1) [3]	OR **B1** for 5 or 1 5 10 10 5 1 row of Pascal's triangle seen	element without evaluation to 5
			Total	3		
6			$(2x)^5 + 5(2x)^4(-3) + 10(2x)^3(-3)^2 + 10(2x)^2(-3)^3$ $+5(2x)(-3)^4 + (-3)^5$ $32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243$	M1(AO2.1) M1(AO1.1) A1(AO1.1) A1(AO1.1) [4]	Binomial coefficents 6 terms in powers of x from 0 to 5 Five terms correct Six terms correct	May be unsimplified eg ${}^5C_2$ or 1 5 10 10 5 1 seen From 5 to 0
			Total	4		
7			$a^5 = 32$ $a = 2$ $10a^2b^3$ [$= -1080$] $4b^3 = -108$ oe $b = -3$	B1 B1 B1 B1 B1	must have evidence that they have considered the constant term **B0** for $a = \pm 2$, but allow them to gain all marks for b if earned may include x^3 on both sides, or $(bx)^3$ on left	NB examiners must use annotation in this part; a tick where each mark is earned is sufficient B0 for eg $10a^2bx^3 = -1080x^3$

				[5]		

and x^3 on right; may have subst their a^2; condone poor notation with inconsistent xs. for subst $a = 2$ in $10a^2b^3 = -1080$ oe if 0 in qn, allow **B1** for 1 5 10 10 5 1 row of Pascal's triangle seen or for $^5C_3 = 10$	B0 for $4b^3 = -108x^3$ etc those trialling factors of –108: Allow up to 3 marks (B0,B1,B1 if earned,B0,B1) for reaching $a = 2$ and $b = -3$ with trialling unless explicit reference to 32 in checking, in which case award up to full marks (in effect explicit reference showing their solution fits both constraints triggers 1st and 4th B1s)

Examiner's Comments

This problem-solving binomial expansion question discriminated extremely well. Some candidates misunderstood the concept of the constant term being 32 and this was then applied incorrectly in a variety of ways, either being assigned to

the value of *a* or to $^5C_3$. Another common error was to work with the term bx^3 rather than $(bx)^3$ sometimes leading to an answer $b = 27$ or –27. Having *x*'s on only one side of an equation and then ignoring them until the last statement was also common, as was a correct $b^3 = -27$ followed by the loss of the negative sign, leading to $b = 3$. Candidates' trialling factors of –108 (with no consideration of the 32) often reached correct values for a and b but were not awarded full marks since this went against the rubric on the front cover which requires that candidates show sufficient detail of the working to indicate that a correct method has been used. However even the

					poorest candidates usually gained a mark for identifying the binomial coefficient 10.	
			Total	5		
8			${}_{15}C_5\left(x^2\right)^5\left(\frac{1}{x}\right)^{10}$ 3003	M1 (AO 3.1a) A1 (AO 1.1) [2]	Identifying term with $\left(x^2\right)^5\left(\frac{1}{x}\right)^{10}$ — Must see **Examiner's Comments** This was a standard question for which many fully correct solutions were provided. Whilst not required, a clear justification of why ${}^{15}C_5$ is needed may help ensure the correct answer of 3003 is obtained, and could gain partial credit if a minor error is made.	
			Total	2		
9			EITHER $(3)^3 + 3(3)^2(-2x) + 3(3)(-2x)^2 + (-2x)^3$ $= 27 - 54x + 36x^2 - 8x^3$ OR $(3-2)^2 = (9 - 12x + 4x^2)$ $(3-2x)(9 - 12x + 4x^2)$	M1 (AO1.1a) M1 (AO1.1b) A1 (AO1.1b) A1 (AO1.1b) [4] M1 M1 A1	Use of Binomial coefficients Powers of 3 and $(-2x)$ Condone no brackets or $(2x)$ used. At least 3 simplified terms correct All correct and simplified Attempting to square	

			$= 27 - 54x + 36x^2 - 8x^3$	A1 [4]	Multiplying their answer by third bracket At least 3 simplified terms correct All correct and simplified **Examiner's Comments** This was well answered by the majority of candidates although a significant number did not correctly use brackets round the $(-2x)$ and so lost a method mark and the accuracy marks. Some also made the mistake of writing $3^3 = 9$. Check numerical answers with a calculator.	
			Total	4		
10			${}_5C_3(2)^2x^3$ $40x^3$	M1 (AO 1.1) A1 (AO 1.1) [2]	For ${}_5C_3(2)^2$ or correct coefficient	
			Total	2		